simplicity. For transitioning into range (A), i.e. $y = 0$:

$$P^0_{xy} = \sum_{k=1}^{c} P_{xk} \quad P_{xy}:$$

For transitioning into range (B), i.e. $0 < y < c$:

$$P^0_{xy} = P_{x;y \quad 1} + P_{x;y+c} \quad (\quad + \quad)P_{xy}:$$

For transitioning into range (C), i.e. $c \leq y < X_r$:

When $y < X_r \quad d$, the transition into y cannot be the result of dividend payment since, after paying dividends, the smallest possible amount of cash is $X_r \quad d$:

$$P^0_{xy} = P_{x;y \quad 1} + P_{x;y+c} \quad (\quad + \quad)P_{xy}:$$

When $y \quad X_r \quad d$, transition into y could have occurred due to a dividend payment:

$$P^0_{xy} = P_{x;y \quad 1} + P_{x;y+c} + P_{x;y+d} \quad (\quad + \quad)P_{xy}:$$

For transitioning into range (D), i.e. $X_r \quad y \quad X_{max}$:

When $y \quad \min(X_{max} \quad c; X_{max} \quad d)$, the transition could have been a result of a claim payment, premium payment, or dividend payment:

$$P^0_{xy} = P_{x;y \quad 1} + P_{x;y+c} + P_{x;y+d} \quad (\quad + \quad + \quad)P_{xy}:$$

When $y > \max(X_{max} \quad c; X_{max} \quad d)$, the transition could only have been a result of a premium payment:

$$P^0_{xy} = P_{x;y \quad 1} \quad (\quad + \quad + \quad)P_{xy}:$$

When $c < d$ and $X_{max} \quad d < y \quad X_{max} \quad c$, the transition could have been a result of a claim payment or a premium payment, but not a dividend payment:

$$P^0_{xy} = P_{x;y \quad 1} + P_{x;y+c} \quad (\quad + \quad + \quad)P_{xy}:$$

When $d < c$ and $X_{max} \quad c < y \quad X_{max} \quad d$, the transition could have been a result of a dividend payment or a premium payment, but not a claim payment:

$$P^0_{xy} = P_{x;y \quad 1} + P_{x;y+d} \quad (\quad + \quad + \quad)P_{xy}:$$

For transitioning into range (E), i.e. $y = X_{max}$:

$$P^0_{xy} = P_{x;y \quad 1} \quad (\quad + \quad)P_{xy}:$$

G) Kolmogorov's backward equations. Similarly, we may state the Kolmogorov's backward equations

$$\frac{@P_{xy}(t)}{@t} = P^0_{xy}(t) = \quad ^1$$

For transitioning out of range (E), i.e. $x = X_{max}$:

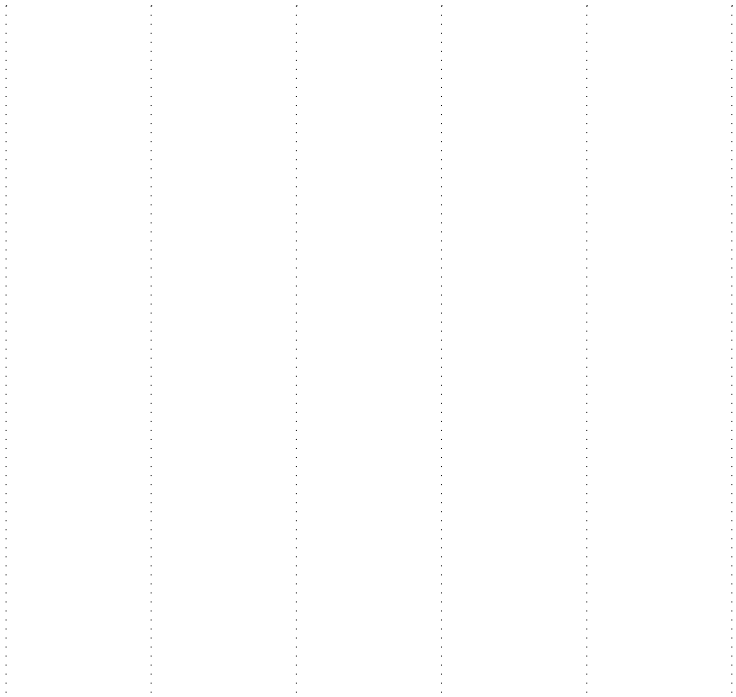
$$P'_{xy} = P_{x\ c;y} + P_{x\ d;y} \quad (\quad + \quad)P_{x;y} :$$

H) Solution of Kolmogorov's equations: A Matlab function is provided to construct the matrix R such that Kolmogorov's forward equations are represented by $P' = RP$:

```
function [R]=Kolmogrov_F(lambda,alpha,beta,c,d,X_r,X_max)

R=zeros(X_max+1); % initialization

% Range A:
```



```
title('pmf of the states between 0 and 5 over quarterly intervals',...
...'Fontsize',14)
axis([0 300 0 0.016])
for t=T
    pmf=expm(R.*t)*p0;
    plot(0:X_max,pmf,'r')
end
```

I) Probability of paying dividends. Finally, we use the solution of Kolmogorov's equation to estimate the probability that the insurance company pays dividends in any given quarter. As previously discussed, dividends can only be paid when the current state is such that $X_r \leq x \leq X_{max}$, i.e. when x is in range (D) or (E). When x is in this range, the expected number of events – either premiums, claims or dividends – is approximately $\lambda_1 + \lambda_2 = 212$. Notice that this is an approximation since we ignore that when x is exactly X_{max} it is impossible to receive payment for a premium. We may divide this number by four to find the expected number of events for each quarter, which is 53. For a given quarter spent entirely in $X_r \leq x \leq X_{max}$

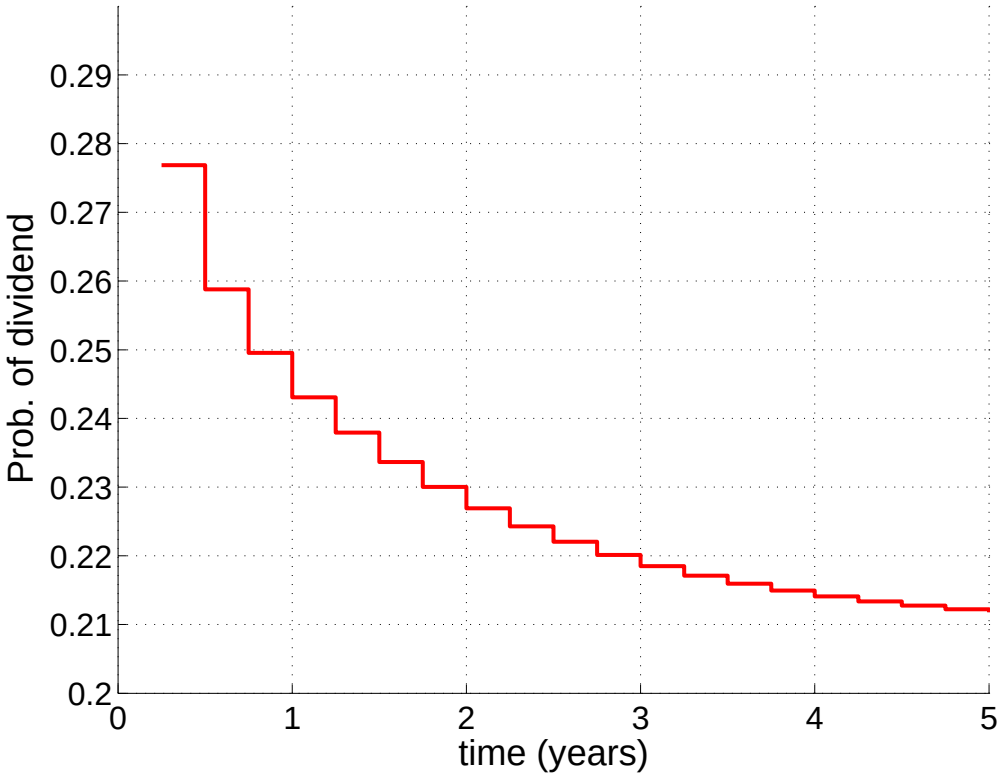


Fig. 5. Approximate probability of paying dividends over quarterly intervals. The probability decreases with time since it is less likely for the company to be over the x_r cash threshold.